

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - x_1^3 - x_2$$

Choose as a candidate for $V(\mathbf{x}) = \frac{1}{2}x_2^2$, with

$$\dot{V} = x_2 \dot{x}_2 = -x_1 x_2 - x_1^3 x_2 - x_2^2$$

The first and second term of the right-hand side are indefinite. But we can integrate them directly, because they are the derivatives of $-\frac{1}{2}x_1^2$ and $-\frac{1}{4}x_1^4$

This leads to $\frac{d}{dt} \left\{ \frac{1}{2}x_1^2 + \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2 \right\} = -x_2^2$

So the Lyapunov function $V(\mathbf{x}) = \frac{1}{2}x_1^2 + \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2$

is positive definite and $\frac{dV}{dt} = -x_2^2$ is positive negative and thus the system is globally asymptotic stable.